Finite Math - Spring 2017

Lecture Notes - 2/10/2017

Homework

- Section 2.6 - 61, 63, 65, 92
- Chapter 2 Review - 96
- Section 3.1 - 9, 11, 15, 18, 20, 22, 24, 26, 34, 50, 55, 58

Section 2.6 - Logarithmic Functions

Example 1. *Solve for x in the following equations:*

(a) $75 = 25e^{-x}$

(b) $42 = 7^{2x+3}$

(c) $200 = (2x-1)^5$

Solution.

(a) $x \approx -1.09861$

(b) $x \approx -0.53961$

(c) $x \approx 1.94270$

Applications. Recall that exponential growth/decay models are of the form

$$A = ce^{rt}.$$

Using the natural logarithm, we can solve for the rate of growth/decay, r, and the time elapsed, t. Let's see this in an example.

Example 2. *The isotope carbon-14 has a half-life (the time it takes for the isotope to decay to half of its original mass) of 5730 years.*

(a) *At what rate does carbon-14 decay?*

(b) *How long would it take for 90% of a chunk of carbon-14 to decay?*

Solution.

(a) *Suppose we have an initial mass of M_0. After half of it decays, the mass will be $\frac{M_0}{2}$ and this happens after $t = 5730$ years has elapsed. Plugging all this into our model, we get*

$$\frac{M_0}{2} = M_0 e^{r(5730)} \Longleftrightarrow \frac{1}{2} = e^{5730r}$$

Applying the natural log to each side gives

$$\ln \frac{1}{2} = \ln e^{5730r}$$

Using properties of logarithms, we have

$$\ln \frac{1}{2} = \ln 2^{-1} = -\ln 2$$

and

$$\ln e^{5730r} = 5730r \ln e = 5730r$$

so that

$$-\ln 2 = 5730r.$$

Solving for r*, we get*

$$r = -\frac{\ln 2}{5730} \approx -0.00012$$

This means that carbon-14 decays at a rate of 0.12% *per year.*

(b) *If the mass of* M_0 *loses* 90% *of its mass, we're looking for the time it takes for only* $0.1M_0$ *to remain. So,*

$$0.1M_0 = M_0 e^{-0.00012t}$$

and canceling the M_0*'s gives*

$$0.1 = e^{-0.00012t}.$$

Hit both sides of this with $\ln$ *to get*

$$\ln 0.1 = \ln e^{-0.00012t} = -0.00012t.$$

Solve for t

$$t = -\frac{\ln 0.1}{0.00012} \approx 19,188.21.$$

So, it would take about $19,188.21$ *years for* 90% *of the original mass to decay.*

Section 3.1 - Simple Interest

Suppose you make a deposit or investment of P dollars or you take out a loan of P dollars. The amount P is called the *principal*.

All of these things have an *interest rate* attached to them, essentially rent on the money, which is paid as *interest*.

Simple Interest. Simple interest is computed as

$$I = Prt$$

where I = interest, P = principal, r = annual simple interest rate (written as a decimal), and t = time in years.

Example 3. *Suppose you deposit* \$2,000 *into a savings account with an annual simple interest rate of* 6%. *How much interest will accrue after* 6 *months?*

Solution. *6 months is 0.5 years, so $t = 0.5$. The interest is 6%, so $r = 0.06$. The principal is $P = 2000$. Plug all this is to get*

$$I = 2000(0.06)(0.5) = 60.$$

So, \$60 would have accrued after 6 months.

Future Value. Often, we might be more curious about how much will be in the account or how much will be owed on the loan after a certain period. This amount is called the *future value*. Another name for principal is *present value*. It is found by simply adding the original investment/loan amount to the interest accrued.

Definition 1 (Future Value)**.**

$$A = P + I = P + Prt$$

and in a simplified form

$$A = P(1 + rt)$$

where A =future value, P =principal/present value, r = annual simple interest rate, t = time in years.

Example 4. *Suppose you take out a* \$10,000 *loan at a simple annual interest rate of* 3.2%. *How much would be due on the loan after* 10 *months?*

Solution. *Principal $P = 10000$*
interest rate $r = 0.032$
10 months = $\frac{10}{12}$ years = $\frac{5}{6}$ years, so $t = \frac{5}{6}$ The future value is then

$$\begin{aligned} A &= 10000\left(1 + (0.032)\left(\frac{5}{6}\right)\right) \\ &\approx 10000(1.027) = \$10,266.67 \end{aligned}$$

Example 5. *You make an investment of* \$3,000 *at an annual rate of* 4.5%. *What will be the value of your investment after* 30 *days? (Assume there are* 360 *days in a year.)*

Solution. \$3,011.25